**Regularized Horseshoe as Spike-Slab Prior**

* **Original Horseshoe Prior (in linear regression)**
  + ,
* **Original Spike and Slab Prior**
  + Spike when , slab when . Often set and get a point mass at origin (like in BVSR)
* **Regularized Horseshoe Prior**
  + , where
  + c is effectively a finite slab width, allows us to control the width
  + The “spike” remains at the pole of the horseshoe
  + The “slab” then becomes a normal with variance
  + The original horseshoe can be obtained when , because this leads to , leading to the original parametrization
  + One suggestion: use t-distribution w/ low degrees of freedom on to ease computation, but retain heavy tails
* With this, the shrinkage weight in from before becomes , where for each predictor
  + Thus still have our pseudo-PIP in this form, should be able to do false discovery rate control
* So far can’t find anyone using greedy algorithms with horseshoe prior – have found modified MCMC that scaled to a ~100,000 predictor GWAS
  + <https://www.jmlr.org/papers/volume21/19-536/19-536.pdf>
* Xiang mixture of normal perspective w/ horseshoe, converging point = spike/slab
* Computational aspects in mixture of normal and horseshoe, can we do approximate/greedy optimization on horseshoe
* Gather existing available packages for horseshoe
* SuSiE paper, variational method w/ horseshoe?